

## 普物會考詳解

基礎:

1. 令
- $40\mu\text{C}$
- 為
- $Q_1$
- 、
- $-50\mu\text{C}$
- 為
- $Q_2$
- 、
- $-4\mu\text{C}$
- 為
- $Q_3$
- ，
- $F_3 = F_{13} + F_{23}$
- ，其中

$$F_{13} = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_3}{r^2} = 9 \times 10^9 \times \frac{40\mu \times 4\mu}{0.2^2} = 36\text{N}$$

$$F_{23} = \frac{1}{4\pi\epsilon_0} \frac{Q_2 Q_3}{r^2} = 9 \times 10^9 \times \frac{50\mu \times 4\mu}{0.3^2} = 20\text{N}$$

$$\underline{F_3 = 56\text{N}}$$

ANS: C

PTS: 2

DIF: Average

- 2.
- $\int E \cdot dA = \frac{Q}{\epsilon_0} \rightarrow EA = \frac{Q}{\epsilon_0} \rightarrow \epsilon_0 E = \frac{Q}{A} = 8.85 \times 10^{-12} \times 200 = \underline{1.77\text{nC/m}^2}$

ANS: B

PTS: 2

DIF: Average

3. 令質子從 A 至 B 花了
- $t$
- 秒，加速度
- $a = \frac{dv}{dt} = \frac{30k}{t}$
- ，A 到 B 距離為
- $\frac{(50k+80k)t}{2} = 65kt$
- ，電場對質子做功
- $W = qV = Fd$

$$qV = 1.6 \times 10^{-19} \times V, Fd = 1.67 \times 10^{-27} \times \frac{30k}{t} \times \frac{(50k+80k)t}{2}$$

可得  $V = 20.35\text{V}$ ，又質子帶正電，受電場的力由高電位向低電位移動，因此  $V_A > V_B$ ， $V_B - V_A$  值為負  $(-20.35\text{V})$ 。

ANS: B

PTS: 2

DIF: Average

4. 電位為電場中某點單位電荷所擁有的能量，電位能為電場中電荷所擁有的能量 (
- $qV = W$
- )。自由的正電荷往電場方向移動，
- 位能減少動能增加
- ，故是往低點
- 位移動
- 。

ANS: A

PTS: 1

DIF: Easy

5. 電容並聯相加，串聯取倒數相加後再取倒數
- $\rightarrow C//C = 2C$
- ，
- $2C$
- 串聯
- $2C = \frac{1}{\frac{1}{2C} + \frac{1}{2C}}$

$$\underline{= 1C}$$

ANS: B

PTS: 1

DIF: Easy

- 6.
- $W_1 + W_2 = \frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2 = \frac{1}{2} \times 15\mu \times 60^2 + \frac{1}{2} \times 25\mu \times 60^2 = \underline{72\text{mJ}}$

ANS: D

PTS: 2

DIF: Average

7. 電偶極矩在電場中偏轉，偶極中的正電荷向電場方向移動，負電荷向電場反方向移動，因此電場作正功，而電偶極偏轉前系統能量較大，偏轉後能量較小，因此位能減少。

ANS: B PTS: 2 DIF: Easy

進階:

1. 令線電荷密度  $\lambda = \frac{q}{l} = \frac{80n}{2} = 40nC/m$ ，又  $dq = \lambda dx$ ， $dE = \frac{dq}{4\pi\epsilon_0(8-x)^2} =$

$$\frac{\lambda dx}{4\pi\epsilon_0(8-x)^2}$$

$$E = \frac{\lambda}{4\pi\epsilon_0} \int_0^2 (8-x)^{-2} dx = \frac{\lambda}{4\pi\epsilon_0} (8-x)^{-1} \Big|_0^2 = \frac{\lambda}{96\pi\epsilon_0} = \underline{15N/C}$$

ANS: B PTS: 3 DIF: Challenging

2. A 與 B 帶同性電，會因斥力而相互遠離，原本 A 與 B 相距  $r = 1m$ ，A 與 B

皆靜止，動能  $K = 0$ ，位能  $U = \frac{5Q^2}{4\pi\epsilon_0 r} =$  總能量  $E$

當 A、B 相互遠離至 3 公尺，位能  $U = \frac{5Q^2}{4\pi\epsilon_0 r} \times \frac{1}{3}$ ，總能量  $E$  不變，動能  $K =$

$$E - U = \frac{5Q^2}{4\pi\epsilon_0 r} - \frac{5Q^2}{4\pi\epsilon_0 r} \times \frac{1}{3} = \frac{5Q^2}{6\pi\epsilon_0 r}$$

又因  $F_{AB} = F_{BA}$  (且質量相同)，A 與 B 以相同加速度往相反方向遠離，兩者初速度均為零，因此速度均相同，動能也相同，因此 B 的動能  $K_B = \frac{1}{2} \times \frac{5Q^2}{6\pi\epsilon_0 r}$

$$= \frac{5 \times (12 \times 10^{-6})^2}{12 \times \pi \times 8.85 \times 10^{-12} \times 1} = \underline{2.16J}$$

ANS: D PTS: 3 DIF: Challenging

3.  $V = -\int E dx = -\int C x^2 dx = -C x^3 / 3$

Ans: D Difficulty: M

### <基礎題型>

8. The current carried by the wire can be calculated using the formula:

$$I = JA$$

where  $I$  is the current,  $J$  is the current density, and  $A$  is the cross-sectional area of the wire.

The cross-sectional area of the wire can be calculated using the formula:  $A = \pi r^2$ , where  $r$  is the radius of the wire

Substituting the given values, we get:

$$A = \pi (0.15 \text{ mm})^2 = \pi (0.00015 \text{ m})^2 = 7.07 \times 10^{-8} \text{ m}^2$$

Now, substituting the given value of current density  $J$ , we get:

$$I = JA = 2.8 \times 10^7 \frac{\text{A}}{\text{m}^2} \times 7.07 \times 10^{-8} \text{ m}^2 = 1.98 \text{ A}$$

9.

Find the diameter:

In series:  $R_{\text{net}} = R_1 + R_2 + R_3 + \dots + R_9$

All wires are identical so resistance will be equal.

$$\text{So } R_{\text{net}} = R + R + R + \dots + R \Rightarrow R_{\text{net}} = 9R$$

$$R = \frac{L}{A}$$

$$A = \pi r^2 \Rightarrow A = \frac{\pi d^2}{4} \quad (r = \frac{d}{2})$$

$$\text{So } R_{\text{net}} = \frac{L \times 4 \times 9}{\pi d^2}$$

3.

$$\text{Now } R_{\text{net}} = \frac{L}{A}$$

Let the diameter of the new wire is  $D$

$$A' = \frac{\pi D^2}{4}$$

$$\text{So } R_{\text{net}} = \frac{L \times 4}{\pi D^2}$$

$$\text{According to the equation, } \frac{L \times 4}{\pi D^2} = \frac{L \times 4 \times 9}{\pi d^2}$$

$$\Rightarrow 9D^2 = d^2$$

$$\Rightarrow 3D = d$$

$$\Rightarrow D = \frac{d}{3}$$

$N$  identical wires each of diameter  $d$  and length  $L$  are connected in series. The combination has the same resistance as a single similar wire of length  $L$  but whose diameter is  $\frac{d}{3}$ .

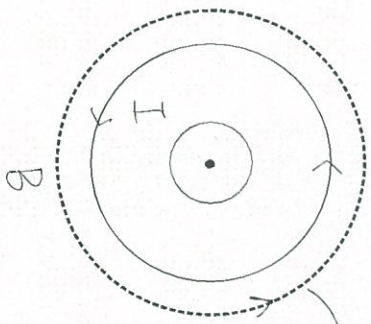
$$11. \quad \vec{F} = -e (\vec{v} \times \vec{B})$$

$$\Rightarrow F = -e [v (\text{north}) \times B (\text{South})]$$

$$= -e v B [\text{north} \times \text{South}]$$

$$= -e v B \sin 180^\circ = 0$$

14.



Dash = Amperian loop

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

 $\cos \theta_0$ 

(both current-carrying wire)

$$\Rightarrow B(2\pi R) = \mu_0 I$$

$$\Rightarrow B = \frac{\mu_0 I}{2\pi R} = \frac{\mu_0 I}{2\pi} \frac{1}{R}$$

&lt;进阶题型&gt;

4.

When the switch  $S$  is open

$$\frac{\text{emf}}{\text{total resistance}} = \text{current}$$

emf ( $E$ ) =  $2 \times (15 + 20) = 70$  Volt, considering internal resistance negligible.when the switch  $S$  is closed then effective resistance becomes

$$R = 15 + \left(\frac{1}{20} + \frac{1}{60}\right)^{-1} = 30 \text{ ohm}$$

$$\text{Then current strength } I = \frac{E}{30} = \frac{70}{30} = \left(2 + \frac{1}{3}\right) \text{ amp}$$

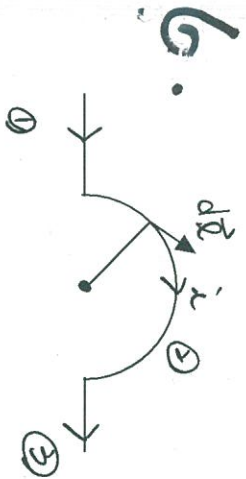
So the ammeter reading will slightly by an amount  $\frac{1}{3}$  ohm

5.

5.

Potential energy of a magnetic dipole in a uniform magnetic field  $U = -\mu B \cos \theta$  clearly the potential energy is maximum when  $\cos = -1$  or  $\theta = \pi$

That is, potential energy is maximum when magnetic dipole with its magnetic moment  $\mu$  is oriented opposite to the direction of magnetic field.



① and ③ don't contribute

$$d\vec{B} = \frac{\mu_0 i dl \times \hat{r}}{4\pi r^2} \sin \theta$$

$$\int_0^{2\pi} \frac{1}{2} dl = L = \pi r$$

$$\Rightarrow B = \frac{\mu_0 i (\pi r)}{4\pi r^2}$$

$$\Rightarrow B = \frac{\mu_0 i}{4r}$$

6.

基礎

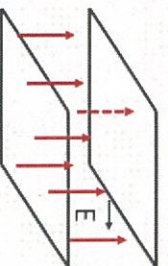
15. A conducting loop with radius of 0.5m. The loop lies in a uniform magnetic field  $B$  that is directed out of the page. The magnitude of the  $B$  is given by  $B=2t^2+3t+2$ , with  $B$  and  $t$  in Tesla and seconds, respectively. What are the induced emf around the loop by the field  $B$  at  $t=10s$ ?

(A) 13.75 V (B) 23.75 V (C) 33.75V (D) 43.75V (E) 53.75V  
(C)

$$E_{\text{ind}} = A(dB/dt) = (3.14 \times 0.5^2)(4t+3) = 0.785 \times (43) = 33.75V$$

16. The magnitude of the electric field between the two parallel plates is  $E = (4 \times 10^5) - (6 \times 10^4 t)$ , with  $E$  in V/m and  $t$  in seconds. At  $t = 0$ ,  $\vec{E}$  pointed upward. The plate area is  $4.0 \times 10^{-2} \text{ m}^2$ . For  $t \geq 0$ , what is the displacement current between the plates? Note:  $\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2$

- (A)  $3.2 \times 10^{-8} \text{ A}$   
(B)  $-3.2 \times 10^{-8} \text{ A}$   
(C)  $-1.6 \times 10^{-8} \text{ A}$   
(D)  $1.6 \times 10^{-8} \text{ A}$   
(E)  $3.6 \times 10^{-8} \text{ A}$



Ans : B

$$i_d = \epsilon_0 \frac{d\Phi_E}{dt} = \epsilon_0 A \frac{dE}{dt} = \epsilon_0 A \frac{d}{dt} [(4.0 \times 10^5) - (6.0 \times 10^4)t] \\ = -\epsilon_0 A (6.0 \times 10^4)$$

$$= -(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(6.0 \times 10^{-2} \text{ m}^2)(6.0 \times 10^4 \text{ V/m} \cdot \text{s})$$

$$= -3.2 \times 10^{-8} \text{ A}$$

17. A 16H inductor carries a current of 8.0 A. At what rate must the current be changed to produce a 24 V emf in the inductor?

(A) -1.5 A/s (B) -3.2 A/s (C) -6.0 A/s (D) -7.1 A/s (E) -9.0 A/s

Ans : A

$$\varepsilon = -L \left( \frac{dI}{dt} \right) \rightarrow 24 \text{ (V)} = -16 \text{ (H)} \left( \frac{dI}{dt} \right) \rightarrow \frac{dI}{dt} = -1.5 \text{ A/s}$$

18. An external alternating emf device, with maximum amplitude of emf  $\mathcal{E}_m = 36\text{V}$  and driving frequency  $f_d = 60\text{Hz}$ , was connected to a series RLC circuit, as shown in Figure, with  $R = 200 \text{ ohm}$ ,  $L = 230\text{mH}$  and  $C = 15\mu\text{F}$ . What is the natural angular frequency of the circuit?

(A) 60 rad/s (B) 18.23 rad/s (C) 538 rad/s (D) 0.147 rad/s (E) 147 rad/s

Ans : C

$$\frac{1}{\sqrt{LC}} = \frac{1}{(230 \times 10^{-3} \times 15 \times 10^{-6})^{0.5}} = 538 \text{ rad/s}$$

19. A plane electromagnetic wave has a maximum electric field magnitude of  $2.50 \times 10^{-5} \text{ V/m}$ . Find the magnetic field amplitude.

(A)  $1.06 \times 10^{-14} \text{ T}$   
 (B)  $2.14 \times 10^{-14} \text{ T}$   
 (C)  $4.32 \times 10^{-14} \text{ T}$   
 (D)  $6.64 \times 10^{-14} \text{ T}$   
 (E)  $8.33 \times 10^{-14} \text{ T}$

Ans : E

$$\text{由於 } B = \frac{E}{c}$$

$$\text{所以 } B = \frac{2.50 \times 10^{-5} \text{ V/m}}{3.00 \times 10^8 \text{ m/s}} \cong 8.33 \times 10^{-14} \text{ T}$$

20. Find the intensity of the electromagnetic wave in an electromagnetic wave with wave length 710 nm and a peak electric field magnitude of 1.76 V/m.

- (A) 0.0041 W/m<sup>2</sup>  
 (B) 0.0062 W/m<sup>2</sup>  
 (C) 0.0083 W/m<sup>2</sup>  
 (D) 0.0126 W/m<sup>2</sup>  
 (E) 0.1126 W/m<sup>2</sup>

Ans : A

$$I = \frac{1}{2} c \epsilon_0 E_0^2 = \frac{1}{2} \left( 3 \times 10^8 \frac{\text{m}}{\text{s}} \right) \left( 8.85 \times 10^{-12} \frac{\text{F}}{\text{m}} \right) \left( 1.76 \frac{\text{V}}{\text{m}} \right)^2 = 0.0041 \text{ W/m}^2$$

進階

7. In an oscillating LC circuit in which  $C = 4.00 \text{ } \mu\text{F}$ , the maximum potential difference across the capacitor during the oscillations is 1.50 V and the maximum current through the inductor is 50.0 mA. What is the magnitude of the inductance L?
- (A)  $5.40 \times 10^{-3} \text{ H}$     (B)  $6.40 \times 10^{-3} \text{ H}$     (C)  $2.70 \times 10^{-3} \text{ H}$   
 (D)  $3.40 \times 10^{-3} \text{ H}$     (E)  $7.40 \times 10^{-3} \text{ H}$

Ans : A

$$\text{由於 } U = \frac{1}{2} LI^2 = \frac{1}{2} \frac{Q^2}{C} :$$

$$L = \frac{1}{C} \left( \frac{Q}{I} \right)^2 = \frac{1}{C} \left( \frac{CV_{\max}}{I} \right)^2 = (6.00 \times 10^{-6} \text{F}) \left( \frac{1.50 \text{V}}{50.0 \times 10^{-3} \text{A}} \right)^2 = 5.40 \times 10^{-3} \text{H}$$

8. In an oscillating series RLC circuit, find the time required for the maximum energy present in the capacitor during an oscillation to fall to half its initial value. Assume  $q = Q$  at  $t = 0$ .

(A)  $\frac{2L}{R} \ln\left(\frac{1}{\sqrt{2}}\right)$  (B)  $\frac{2L}{R} \ln 2$  (C)  $\frac{L}{R} \ln 2$  (D)  $\frac{L}{\sqrt{2}R} \ln 2$  (E)  $\frac{L}{2R} \ln 2$

Ans : C

令初始值能量一半的電量為  $q_{\text{half}}$ ，此時， $\frac{q_{\text{half}}}{2C} = \frac{1}{2} \frac{Q^2}{2C} \rightarrow q_{\text{half}} = \frac{Q}{\sqrt{2}}$

$$\text{又 } q_{\text{half}} = Qe^{-\frac{Rt}{2L}} \rightarrow \ln\left(\frac{q_{\text{half}}}{Q}\right) = -\frac{Rt}{2L}$$

$$\begin{aligned} \text{將 } q_{\text{half}} = \frac{Q}{\sqrt{2}} \quad \text{代 入} : \quad t &= -\frac{2L}{R} \ln\left(\frac{q_{\text{half}}}{Q}\right) = -\frac{2L}{R} \ln\left(\frac{1}{\sqrt{2}}\right) = \\ &= -\frac{2L}{R} \ln(2)^{-\frac{1}{2}} = \frac{L}{R} \ln 2 \end{aligned}$$

9. As a parallel-plate capacitor with circular plates 20 cm in diameter is being charged, the current density of the displacement current in the region between the plates is uniform and has a magnitude of 20 A/m<sup>2</sup>. Calculate the magnitude B of the magnetic field at a distance  $r = 50$  mm from the axis of symmetry of this region.

(A) 0.21  $\mu\text{T}$  (B) 2.1  $\mu\text{T}$  (C) 21  $\mu\text{T}$  (D) 6.3  $\mu\text{T}$  (E) 0.63  $\mu\text{T}$

Ans : E

$$\text{由於 } \int \mathbf{B} \cdot d\mathbf{s} = \mu_0 I_{\text{enclosed}} :$$

$$B = \frac{\mu_0 I_{\text{enclosed}}}{2\pi r} = \frac{\mu_0 (I_d \pi r^2)}{2\pi r} = \frac{1}{2} \mu_0 I_d r$$

$$= \frac{1}{2} \left( 1.26 \times 10^{-6} \frac{\text{H}}{\text{m}} \right) \left( 20 \frac{\text{A}}{\text{m}^2} \right) (50 \times 10^{-3} \text{ m}) = 6.3 \times 10^{-7} \text{ T}$$

10. In the previous question, calculate  $dE/dt$  in this region.

(A)  $2.3 \times 10^{13} \frac{\text{V}}{\text{m}\cdot\text{s}}$  (B)  $4.6 \times 10^{13} \frac{\text{V}}{\text{m}\cdot\text{s}}$  (C)  $2.3 \times 10^{12} \frac{\text{V}}{\text{m}\cdot\text{s}}$

(D)  $6.9 \times 10^{12} \frac{\text{V}}{\text{m}\cdot\text{s}}$  (E)  $69 \times 10^{12} \frac{\text{V}}{\text{m}\cdot\text{s}}$

Ans : C

$$\text{由 } i_d = J_d \pi r^2 = \epsilon_0 \frac{d\Phi_E}{dt} = \epsilon_0 \pi r^2 \frac{dE}{dt} :$$

$$\frac{dE}{dt} = \frac{J_d}{\epsilon_0} = \frac{20 \text{ A/m}^2}{8.85 \times 10^{-12} \text{ F/m}} = 2.3 \times 10^{12} \frac{\text{V}}{\text{m}\cdot\text{s}}$$